CSCB63 TUT2 WEEK12
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https://snoopysnipe.github.io/ta/b63f20/

(1) Monty Hall Problem

Monty Hall!

There are 3 doors. Behind one door is the prize, behind the other two doors are empty (or goats, whatever). First you choose one door. Then the game host opens a door, but not what you choose, and not the prize door (the game host knows). The game host now offers you the choice of staying with your first choice or switching to the remaining door.

(a) What is the probability that the prize is behind your first choice?

(b) What is the probability that the prize is behind the remaining door?

Solution: Label the doors #1, #2, and #3. Assume the prize is behind door #1 without loss of generality (can redo with #2 and #3).
You're equally likely to choose any of the doors.

Case 1: Choose #1. It has prize. Host opens #2 or #3, neither has prize.

Case 2: Choose #2. Host opens #3. Remaining door has prize.

Case 3: Choose #3. Host opens #2. Remaining door has prize.

Each case occurs with probability 1/3. Only one case where prize is behind first choice ($p = \frac{1}{3}$).
Two cases where prize is behind remaining door ($p = \frac{2}{3}$).

(2) Birthday Paradox

The Assignment Due Date Paradox

This is a ripoff of the Birthday Paradox but mine is more relevant to students!

You're taking 4 courses. All four Assignment 1's are due in week 4. Each prof independently chooses one weekday (Monday to Friday) for the due date.

(a) How many ways are there overall?

(b) How many ways are there such that all 4 assignments are due on different days, i.e., no two assignments are due on the same day? And so what is the probability that this happens?

(c) How many ways are there such that at least two assignments are due on the same day? And probability?

a) $5^4 = 625$

b) $5 \times 4 \times 3 \times 2 = 120$ probability: $\frac{120}{625} \approx 0.192$

c) $625 - 120 = 505$ probability: $\frac{505}{625} \approx 0.808$

3 The i^{th} Ball

The ith Ball

This is from a past assignment, modified with smaller parameters.

There are 6 red balls and 9 blue balls in a bag; when you draw a ball from the bag, each ball in the bag is equally likely drawn. Randomly draw 3 balls from the bag without replacement---after a ball is drawn, do not put it back into the bag. Find the probability that the ith ball, (1 <= i <= 3), is one of the red balls.

$6/15 = 2/5$. It is just the proportion of red balls, does not care what i is.

Why?

6 ways to make i^{th} ball red. Then 14P2 ways to choose other 2 balls,

$= 6 \cdot 14P2$

$= 6 \cdot 14 \cdot 13$

Total permutations of 3 balls out of 15 balls

$= 15P3$

$= 15 \cdot 14 \cdot 13$

To get probability, just take ratio:

$$\frac{6\cdot\cancel{14}\cdot\cancel{13}}{15\cdot\cancel{14}\cdot\cancel{13}} = \frac{6}{15}$$

Can also just think of it as choosing ith ball as red (which happens with probability $6/15$) and then dont care about how the other 2 balls are chosen.