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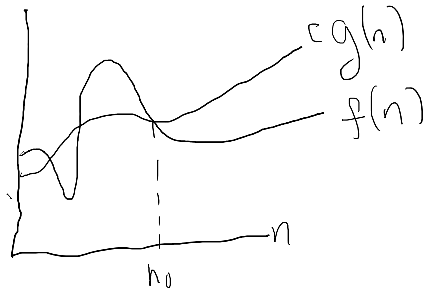
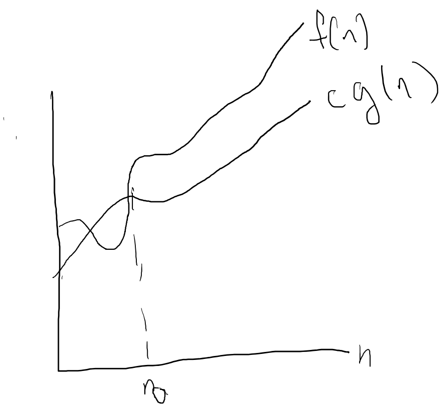
Asymptotic Upper Bound (O-notation)

$O(g(n)) = \{f(n)$: there exists positive constants c and n_0 such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0\}$



Asymptotic Lower Bound (Ω-notation)

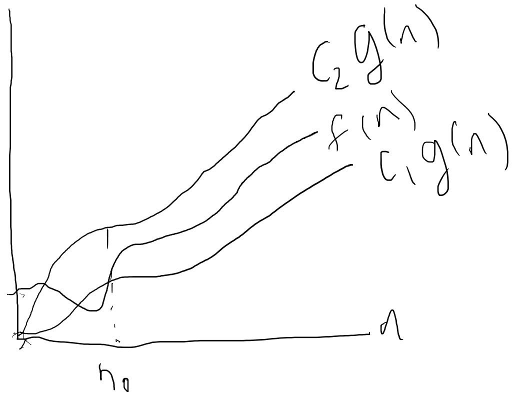
$\Omega(g(n)) = \{f(n)$: there exists positive constats c and n_0 such that $0 \leq cg(n) \leq f(n)$ for all $n \geq n_0\}$



Asymptotic Tight Bound (Θ-notation)

$\Theta(g(n)) = \{f(n)$: there exists positive constants c_1, c_2, and n_0 such that

$0 \le c_1 g(n) \le f(n) \le c_2 g(n)$
for all $n \ge n_0$ }



ex: $f(n) = 12n^2 + 14n + 10$ WTP: $f(n) \in O(n^2)$

$\forall n \ge 1$

$\le 12n^2 + 14n^2 + 10$

$= 26n^2 + 10$

$\forall n \ge 10$

$\le 26n^2 + n$

$\le 26n^2 + n^2$

$= 27n^2$

$\therefore f(n) \in O(n^2)$ since $\exists\ c > 0$ (27) and n_0 (10)

such that $0 \le f(n) \le cn^2$ $\forall n \ge n_0$

ex: $f(n) = \frac{1}{2}n^2 - 3n$ WTP: $f(n) \in \Theta(n^2)$

we have

$c_1 n^2 \le \frac{1}{2}n^2 - 3n \le c_2 n^2$

$\forall n \geq 1$

$$c_1 \leq \underbrace{\frac{1}{2} - \frac{3}{n}} \leq c_2$$

since $n \geq 1$ choose $\boxed{c_2 = \frac{1}{2}}$

Remember $0 \leq c_1 \leq \frac{1}{2} - \frac{3}{n} \leq c_2$

$\Rightarrow n \geq 6$

but $c_1 > 0$, so $n > 6$

try $n = 7$

$$\frac{1}{2} - \frac{3}{7}$$

$$= \frac{7}{14} - \frac{6}{14} = \frac{1}{14}$$

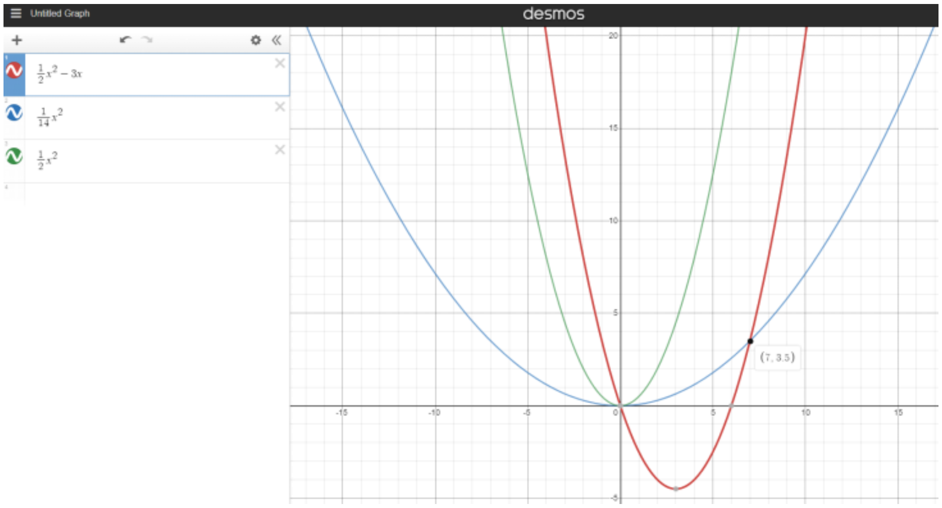
Let $\boxed{c_1 = \frac{1}{14}, \; n_0 = 7}$

$\therefore \frac{1}{2}n^2 - 3n \in \Theta(n^2)$ because $\exists c_1 > 0 \; (\frac{1}{14})$, $c_2 > 0 \; (\frac{1}{2})$ and $n_0 \; (7)$ such that $0 \leq c_1 n^2 \leq \frac{1}{2}n^2 - 3n \leq c_2 n^2 \; \forall n \geq n_0$

ex: Prove $12n^2 + 14n + 10 \in \Omega(n^2)$

choice of c and n_0 is trivial

$\Rightarrow 12n^2 + 14n + 10 \in \Theta(n^2)$



Untitled Graph
desmos
$\frac{1}{2}x^2 - 3x$
$\frac{1}{14}x^2$
$\frac{1}{2}x^2$
(7, 3.5)