CSCB63 TUT2 WEEK5
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https://snoopysnipe.github.io/ta/b63f20/

A = [9, 7, 29, 16, 2, 14, 47, 4]

i = 1
t = 7
j = 1
A = [9, 9, 29, 16, 2, 14, 47, 4]
j = 0
A = [7, 9, 29, 16, 2, 14, 47, 4]

i = 2
t = 29
j = 2

i = 3
t = 16
j = 3
A = [7, 9, 29, 29, 2, 14, 47, 4]
j = 2
A = [7, 9, 16, 29, 2, 14, 47, 4]

i = 4
t = 2
j = 4
A = [7, 9, 16, 29, 29, 14, 47, 4]

j=3

A=[7, 9, 16, 16, 29, 14, 47, 4]

j=2

A=[7, 9, 9, 16, 29, 14, 47, 4]

j=1

A=[7, 7, 9, 16, 29, 14, 47, 4]

j=0

A=[2, 7, 9, 16, 29, 14, 47, 4]

i=5

t=14

j=5

A=[2, 7, 9, 16, 29, 29, 47, 4]

j=4

A=[2, 7, 9, 16, 16, 29, 47, 4]

j=3

A=[2, 7, 9, 14, 16, 29, 47, 4]

i=6

t=47

j=6

A=[2, 7, 9, 14, 16, 29, 47, 4]

i = 7
t = 4
j = 7
A = [2, 7, 9, 14, 16, 29, 47, 47]
j = 6
A = [2, 7, 9, 14, 16, 29, 29, 47]
j = 5
A = [2, 7, 9, 14, 16, 16, 29, 47]
j = 4
A = [2, 7, 9, 14, 14, 16, 29, 47]
j = 3
A = [2, 7, 9, 9, 14, 16, 29, 47]
j = 2
A = [2, 7, 7, 9, 14, 16, 29, 47]
j = 1
A = [2, 4, 7, 9, 14, 16, 29, 47]

```
def IS (A):
  i = 1
  while (i < len(A))
    t = A[i];
    j = i;
    while (j > 0 AND A[j-1] > t)
      A[j] = A[j-1];
      j = j-1;
    A[j] = t;
    i = i+1;
```

z, y, x

In-place insertion sort worst case complexity analysis:

Step count:

x = 6 steps (inner loop body)
y = 6 steps (inner loop test)
z = 10 steps (outer loop body not including x and y + outer loop test)

$\#steps_{wc}$ occurs when input list is in strictly decreasing order

$$\#steps_{wc} = \{[1(x+y)+z]+[2(x+y)+z]+\ldots+[(n-1)(x+y)+z]\}$$

$$= (x+y)(1+2+\ldots+(n-1)) + z(n-1)$$

$$= (x+y)\sum_{i=1}^{n-1} i + z(n-1)$$

$$= (x+y)\frac{n(n-1)}{2} + z(n-1)$$

<u>Upper Bound</u>

$$\#steps_{wc} = (x+y)\frac{n(n-1)}{2} + z(n-1)$$

$$< (x+y)n^2 + zn^2$$

$$= \underbrace{(x+y+z)}_{c}n^2$$

$$= cn^2 \qquad \text{where} \quad c = x+y+z$$

$\therefore$ IS $\in O(n^2)$

<u>Lower Bound</u>

want $\#steps_{wc} > dn^2$ for some d

when $n=2$ $\#steps_{wc} = (x+y)\frac{n(n-1)}{2} + z(n-1)$

$$= (x+y)\frac{2(1)}{2} + z(2-1)$$

$$= (x+y)+z$$

$$= 22$$

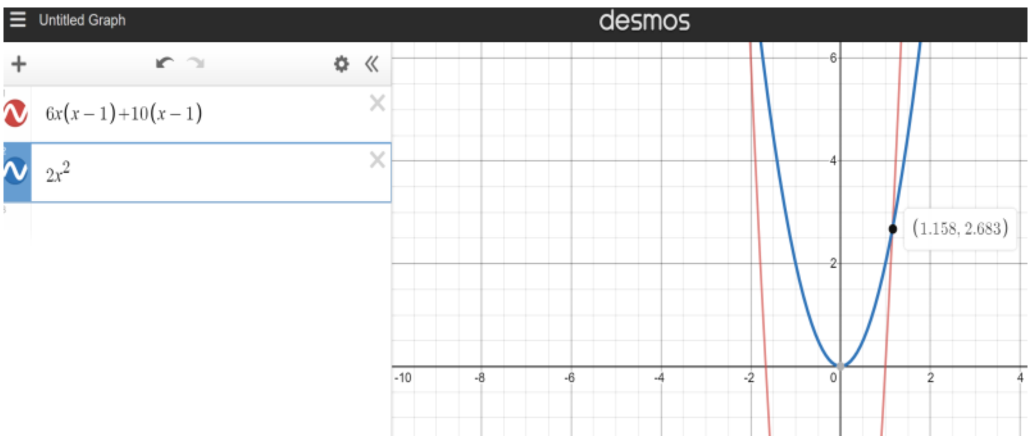
we have $22 > dn^2$

$\Rightarrow 22 > 4d$

$\Rightarrow \frac{22}{4} > d$

Therefore for $n \geq 2$, $d = 2$ $\quad \#steps_{wc} > dn^2$

$\therefore$ IS $\in \Omega(n^2)$



Additional notes:

$$\#steps_{wc} = (x+y)\frac{n(n-1)}{2} + 2(n-1)$$

$$= \left(\frac{x+y}{2}\right)n^2 + \left(2 - \frac{x+y}{2}\right)n - 2$$

$$> dn^2$$

$$\Leftrightarrow d < \frac{1}{n^2}\left[\left(\frac{x+y}{2}\right)n^2 + \left(2 - \frac{x+y}{2}\right)n - 2\right]$$

Consider $\lim_{n \to \infty} \frac{1}{n^2}\left[\left(\frac{x+y}{2}\right)n^2 + \left(2 - \frac{x+y}{2}\right)n - 2\right]$

$$= \frac{x+y}{2}$$

$$= 6$$

So as $n \to \infty$, $d < 6$

and recall earlier we said when $n=2$, $d < \frac{22}{4}$

Thus, its ok to choose $d=2$ for $n \geq 2$