CSCB63 TUT2 WEEK7
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https://snoopysnipe.github.io/ta/b63f20/

① Using limits to prove O

Assume $\exists_{n_0}, \forall_{n \geq n_0}, f(n) > 0$ and $g(n) > 0$

Thm: If $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ exists and is finite,

then $f(n) \in O(g(n))$

ex: Prove $\frac{n(n+1)}{2} \in O(n^2)$

$$\lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^2} = \frac{1}{2}$$

$\therefore \frac{n(n+1)}{2} \in O(n^2)$

ex: Prove $\ln(n) \in O(n)$

l'hopital's rule

$$\lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$$

$\therefore \ln(n) \in O(n)$

Thm: If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$, $f(n) \notin O(g(n))$

ex: disprove $n^2 \in O(n)$

$$\lim_{n\to\infty} \frac{n^2}{n} = \lim_{n\to\infty} n = \infty$$

$\therefore n^2 \notin O(n)$

ex: disprove $n \notin O(\ln(n))$

$$\lim_{n\to\infty} \frac{n}{\ln(n)} = \lim_{n\to\infty} \frac{1}{\frac{1}{n}} = \lim_{n\to\infty} n = \infty$$

$\therefore n \notin O(\ln(n))$

If $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ DNE and not ∞,

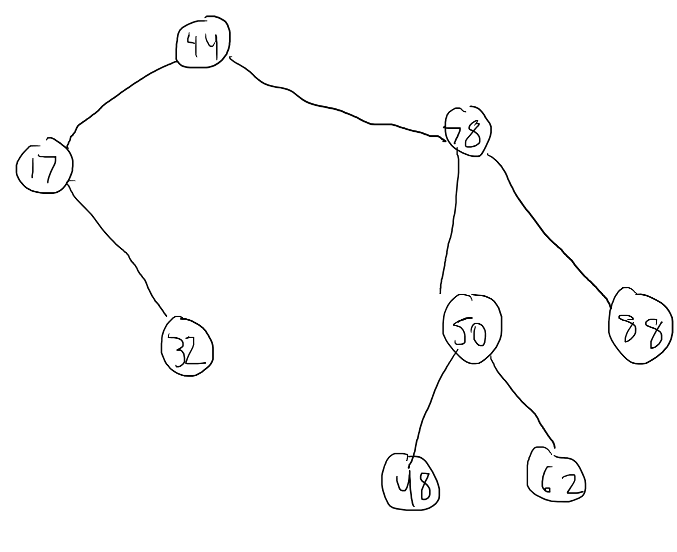
no conclusion

2) AVL Trees

↳ BST

↳ but at every node, subtree heights differ by at most 1

ex:

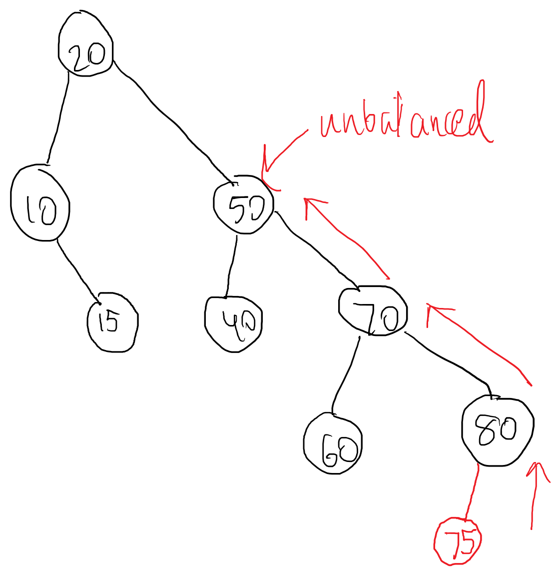


lookup

insert } need to rebalance

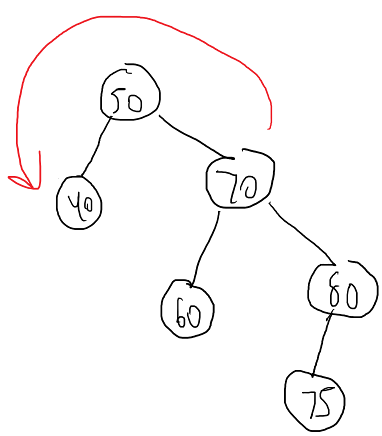
delete }

ex:

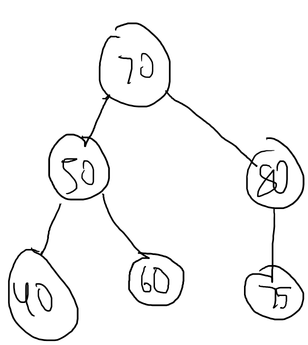


insert 75

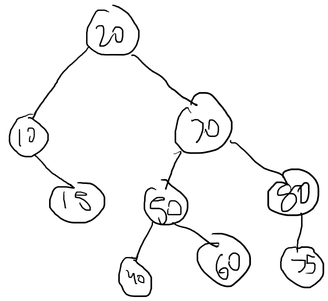
"single rotation counter clockwise"



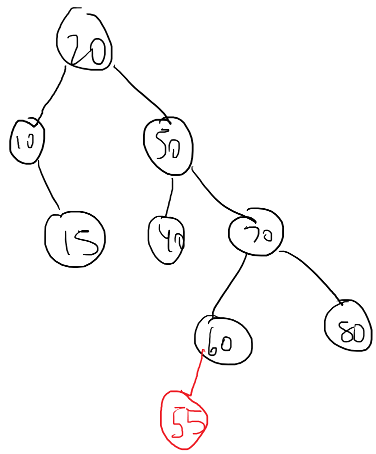
⇒



$\Rightarrow$

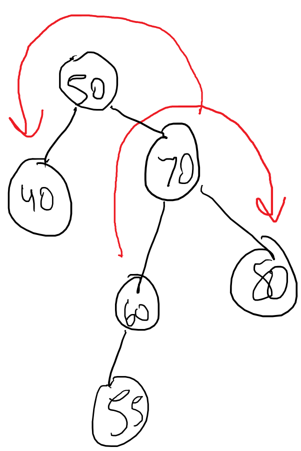


ex:

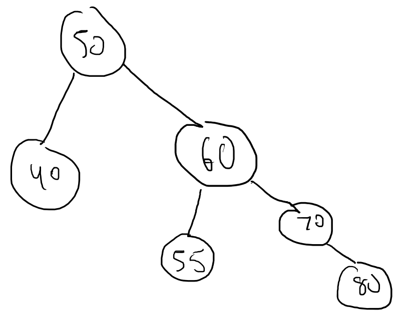


insert 55

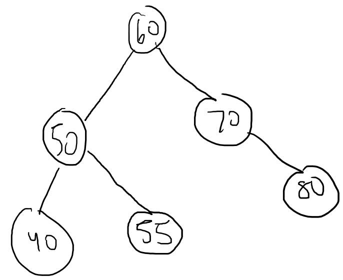
"double rotation clockwise then counterclockwise"



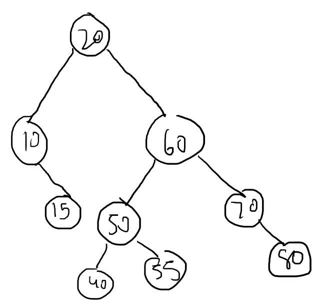
$\Rightarrow$



$\Rightarrow$



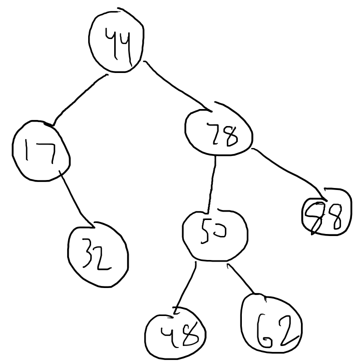
$\Rightarrow$



mirror cases for left side

delete: ① leaf (eg: 32, 48, 62, 88), just unlink
② parent of 1 (eg: 17), parent adopts
③ parent of 2 (eg: 44), find successor, replace, parent of successor adopts successor's right child

ex:



to find successor: ① go right down 1
② go all the way left

then rebalance starting from the adopter, then up