

1. Charge append \$4
 normal append uses \$1 save \$3

n is array size

m is # elements in array

When $m=n$, create new array of size $1.5n$

New array has $n/3$ empty slots

Moving all elements over costs m

After another $n/3$ appends array is full again, since saved \$3 per append, we still have at least \$ m savings. Which is enough to move them over again.

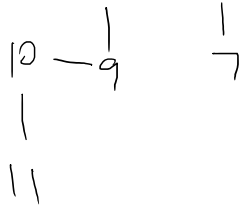
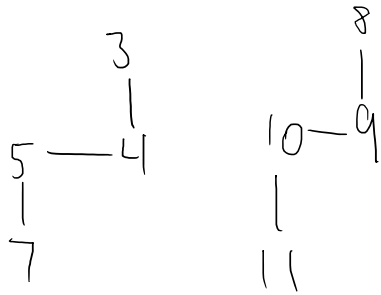
So \$4 is sufficient to cover the costs

So amortized complexity of append is $O(1)$.

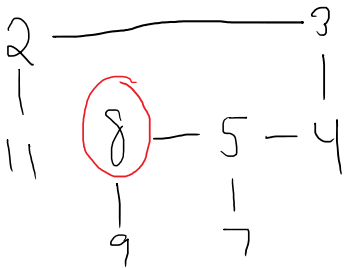
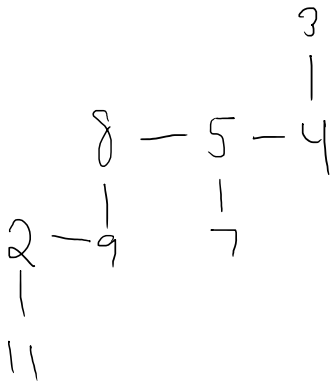
2. a) $\overset{\min}{\downarrow}$ 2 - 3 - 4 - 5 - 7 - 8 - 9 - 10 - 11

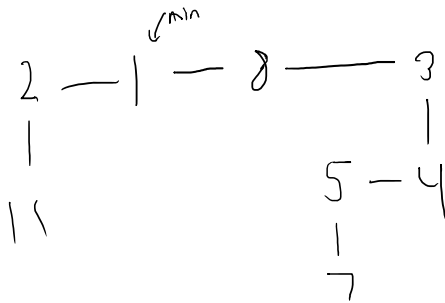
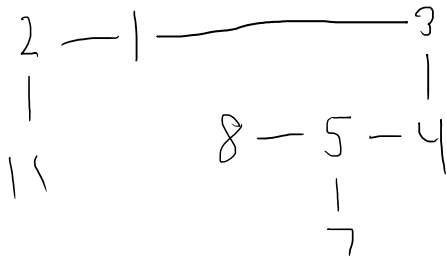
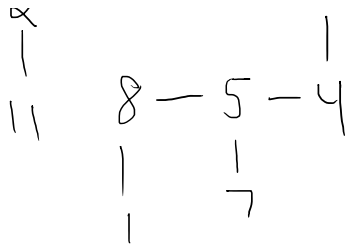
b) $\begin{array}{cccc} 3 & 5 & 8 & 10 \\ | & | & | & | \\ 4 & 7 & 9 & 11 \end{array}$

4 7 9 11



c)





3. def has-route(i, j):
 for $i = 1$ to n
 make-set(c_i)
 for each $(c_i, c_j) \in R$:
 Union(c_i, c_j)
 return find(c_i) == find(c_j)

4. Since the only difference is path compression, need a find operation. Path compression is only useful when finding a node that is

at least 2 nodes away from root.

To make a tree of height 2, need to union 2 2-node trees.

eg: make(1), make(2), make(3), make(4)

1 2 3 4

union(1,2) union(3,4)

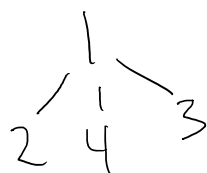


Union (1,3)



← without path compression

now find deepest node, find (4)



← with path compression

1. 8