CSCB63 TUT4 WEEK2
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https://snoopysnipe.github.io/ta/b63w21/

HANDOUT:
https://mathlab.utsc.utoronto.ca/bretscher/b63/tutorials/w2.pdf

	$\ln(n)$	$\lg(n)$	$\lg(n^2)$	$(lgn)^2$	n	$n*lg(n)$	2^n	$2^{(3n)}$
$\ln(n)$	Y	Y	Y	Y	Y	Y	Y	Y
$\lg(n)$	Y	Y	Y	Y	Y	Y	Y	Y
$\lg(n^2)$	Y	Y	Y	Y	Y	Y	Y	Y
$(\lg n)^2$				Y	Y	Y	Y	Y
n					Y	Y	Y	Y
$n*\lg(n)$						Y	Y	Y
2^n							Y	Y
$2^{(3n)}$								Y

(Remember that lg means log base 2.)

In each cell, fill in "Y" iff (its row function)$\in O$(its column function).

Fill in proofs for selected big-O cells:

1. $n \in O(n\lg(n))$, using the definition of big-O:
2. $n\lg(n) \notin O(n)$ using the definition of big-O: (Note: This can be explained as a proof by contradiction...other ways possible too).
3. $2^{(3n)} \notin O(2^n)$, using a limit theorem from lecture (you may not have seen the limit theorem if your tutorial is before the Wed. class).

Some practice questions:

1. $6n^5 + n^2 - n^3 \in \Theta(n^5)$
2. $3n^2 - 4n \in \Omega(n^2)$

1. WTS: $\exists c, n_0$ such that $0 \leq n \leq cn\lg(n)$ $\forall n \geq n_0$

<u>Intuition:</u> Note that $\lg(2) = 1$, so
$n \leq n\lg(n)$ $\forall n \geq 2$ since
$\lg(n)$ only increases

<u>Formally:</u> Choose $n_0 = 2$, $C = 1$
Since $n \geq n_0 = 2$
$2 \leq n$
lg both sides
$\Rightarrow 1 \leq \lg n$

Now consider

$$
\begin{aligned}
& n \\
&= n \cdot 1 \\
&\leq n \cdot \lg(n) \\
&= c \cdot n \cdot \lg(n)
\end{aligned}
$$

and clearly $0 \leq n$ $\blacksquare$

2. Proof by contradiction

Suppose $n \lg n \in O(n)$

$\Rightarrow \exists_{c, n_0}$ st $0 \leq n \lg n \leq cn \quad \forall n \geq n_0$

Change the condition

$\forall n \geq n_0$ to $\forall n \geq \max(n_0, 1)$

↳ still true since $\max(n_0, 1) \geq n_0$

Now $\forall n \geq \max(n_0, 1)$

$$
\begin{aligned}
n \lg n &\leq cn \\
\lg n &\leq c \qquad \text{since } n \geq 1 \\
n &\leq 2^c
\end{aligned}
$$

Now choose $n = \max(n_0, 1, 2^c + 1)$, then $n \geq \max(n_0, 1)$ and $n \leq 2^c$. However this $n > 2^c$ since $2^c \geq n \geq n_0 \geq 1$.

CONTRADICTION

3. Use limit theorem:

$$\lim_{n \to \infty} \frac{2^{(3n)}}{2^n}$$

$$= \lim_{n \to \infty} 2^{(3n-n)}$$

$$= \lim_{n \to \infty} 2^{2n}$$

$$= \infty$$

$$\therefore \quad 2^{(3n)} \notin O(2^n)$$

PRACTICE:

1. WTS: $\exists c_1, c_2, n_0$ st $0 \leq c_1 n^5 \leq 6n^5 + n^2 - n^3 \leq c_2 n^5$

$\forall n \geq n_0$

Big-O

$$6n^5 + n^2 - n^3$$
$$\leq 6n^5 + n^2$$
$$\leq 6n^5 + n^5$$
$$= 7n^5$$

$\forall n \geq 1$

Big-Ω

$$6n^5 + n^2 - n^3$$
$$\geq 6n^5 - n^3$$
$$\geq 6n^5 - n^5$$
$$= 5n^5$$

$\Rightarrow c_1 = 5,\ c_2 = 7,\ n_0 = 1$

2. WTS: $\exists c, n_0$ st $0 \le cn^2 \le 3n^2 - 4n \quad \forall n \ge n_0$

$$cn^2 \le 3n^2 - 4n$$
$$cn \le 3n - 4 \qquad \forall n \ge 1$$
$$4 \le (3-c)n$$
$$\frac{4}{3-c} \le n \Rightarrow \text{Let } c=1 \Rightarrow n \ge 2 \Rightarrow n_0 = 2$$

Let $c = 2 \Rightarrow n \ge 4 \Rightarrow n_0 = 4$

$3n^2 - 4n$

$\ge 3n^2 - 2n^2 \quad \forall n \ge 2$

$= n^2$

$\Rightarrow c = 1,\ n_0 = 2$

Additional Notes:

Recall logarithms are related to each other by some constant thanks to the change of base formula: $\log_b(x) = \log_c(x) / \log_c(b)$ Which is why we were able to easily conclude that lg(n) in O(ln(n)) and ln(n) in O(lg(n)).