CSCB63 TUT4 WEEK9
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https://snoopysnipe.github.io/ta/b63w21/

The Assignment Due Date Paradox

You're taking 4 courses. All four Assignment 1's are due in week 4. Each prof independently chooses one weekday (Monday to Friday) for the due date.
(a) How many ways are there overall?
(b) How many ways are there such that all 4 assignments are due on different days, i.e., no two assignments are due on the same day? And so what is the probability that this happens?
(c) How many ways are there such that at least two assignments are due on the same day? And probability?

a) $5^4 = 625$

b) $5 \times 4 \times 3 \times 2 = 120$

$\Rightarrow$ probability: $\frac{120}{625} \approx 0.192$

c) $625 - 120 = 505$

$\Rightarrow$ probability: $\frac{505}{625} \approx 0.808$

The ith Ball

There are 6 red balls and 9 blue balls in a bag; when you draw a ball from the bag, each ball in the bag is equally likely drawn. Randomly draw 3 balls from the bag without replacement---after a ball is drawn, do not put it back into the bag. Find the probability that the ith ball, (1 <= i <= 3), is one of the red balls.

Method 1:

6 ways to make the i-th ball red. Then $14P2$ ways to choose other 2 balls.

$= 6 \cdot 14P2$

$= 6 \cdot 14 \cdot 13$

Total permutations of 3 balls out of 15 balls

$= 15P3$

$= 15 \cdot 14 \cdot 13$

$\Rightarrow$ probability: $\frac{6 \cdot 14 \cdot 13}{15 \cdot 14 \cdot 13} = \frac{6}{15}$

Method 2:

Choose/fix i-th ball to be red (which

happens with probability 6/15) and then we don't care about how the other 2 balls are chosen.

Method 3: Take the complement

(To answer a student's question, in our discussion we were looking at the complement incorrectly. The complement is "what is the probability that the i-th ball is not red" ⇒ "what is the probability that the i-th ball is blue")

This is $9/15$ and $1 - 9/15 = 6/15$